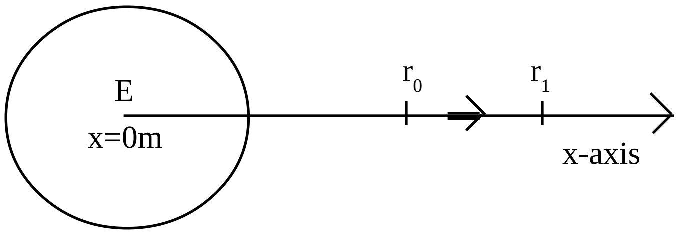
GRAVITATIONAL POTENTIAL ENERGY

What is the work done by the force of gravity in moving an apple of mass, m_A, from a position r_0 (from the center of the earth) to a position r_1 (from the center of the earth)? (i.e., $W_{F_g} = ?$)



Integral Calculus

THM/ $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

ex/ $\int x^5 dx =$ ________?

ex/ $\int_{x_0}^{0} -kx\, dx = -k\frac{x^2}{2}\Big|_{x_0}^{0} =$ __________?

ex/ $\int \frac{1}{x^2} dx =$ ____________?

$$W_{F_g} = \int_{x=r_0}^{x=r_1} F(x)dx = \int_{r_0}^{r_1} -\frac{Gm_E m_A}{x^2}dx$$

$$= -Gm_E m_A\left(-\frac{1}{x}\right)\Big|_{r_0}^{r_1}$$

$$= Gm_E m_A\left[\frac{1}{r_1} - \frac{1}{r_0}\right]$$

What is the work done by the force of gravity in moving the apple from r_0 to infinity(∞)?

$$W_{F_g} = Gm_E m_A\left[\frac{1}{\infty} - \frac{1}{r_0}\right] = -G\frac{m_E m_A}{r_0}$$

If $U_\infty = 0J$ then we are considering "deep space" or infinity to be our zero reference level or point, then

$$U_g = -G\frac{m_E m_A}{r} \text{ for our apple in the earth's } \vec{g}\text{-field.}$$

Escape Velocity

How much speed (v_0) do we need to give our apple in order to just send it to deep space (∞) where $F_g \approx 0N$?

Let's use: $E_0 = E_f$

[Here our starting point is $r_0 = R_E$, $R_E = 6.38\times10^6 m$]

$$K_0 + U_0 = K_f + U_f$$

[Here $K_f = 0J$ since $v_f = 0$, and $U_\infty = 0J$]

$$\frac{1}{2}m_A v_0^2 + -G\frac{m_E m_A}{R_E} = 0 + 0$$

$$v_0 = v_{escape} = \sqrt{\frac{2Gm_E}{R_E}}$$

[Where $G = 6.67\times10^{-11}$ and $m_E = 5.98\times10^{24} kg$.]

________ $\frac{m}{s}$

(continued from p.1)

If c = $3.00 \times 10^{8}\ {}^{m}\!/\!{}_{s}$ and we could crunch the earth into a small sphere, find the new earth radius which would turn the earth into a "black hole". Hypothetical earth radius = ________ m ?

Suppose we want to launch an apple satellite into orbit at a distance, $r_f = 12.8 \times 10^{6}\, m$ from the center of the earth.
What is the necessary orbital speed (v_f)?

$$F_c = m_A a_c = m_A \frac{v^2}{r}$$

$$F_c = F_g = G\frac{m_E m_A}{r^2} = m_A \frac{v_f^2}{r}$$

$$\Rightarrow v_f = \_\_\_\_\_\_\_\_\_\_\_\_ m/s$$

Suppose the necessary orbital speed is $v_f = 6000\ {}^{m}\!/\!{}_{s}$.
What takeoff speed, v_0, is necessary to place the apple in orbit.
Here $r_0 = R_E = 6.38 \times 10^{6}\, m$ and $r_f = 2R_E = 12.8 \times 10^{6}\, m$.

Use $E_0 = E_f$

$$\frac{1}{2} m_A v_o^2 + -G\frac{m_E m_A}{r_0} = \frac{1}{2} m_A v_f^2 + -G\frac{m_E m_A}{r_f}$$

$$v_0 = \_\_\_\_\_\_\_\_\ m/s$$