UNIVERSITY of GUELPH

DEPARTMENT OF PHYSICS
INTEGRATED LABORATORY

Gyroscopic Motion

PURPOSE:

To investigate precession and nutation in a gyroscope.

REFERENCES:

1. *Analytical Mechanics, 6th ed.*, G.R. Fowles and G.L. Cassiday, Saunders, 1999.

2. *Instruction Manual and Experiment Guide for the PASCO Scientific Model ME-8960*, PASCO Scientific, 1997.

INTRODUCTION:

Precession

The "free gyroscope" is an object (usually symmetrical) which is suspended in gimbals or in some other manner such that it is free to rotate about any axis. If such an object is set in rotation about some axis with angular velocity **ω**, and if no torques act on it which are not in the direction of **ω**, its rotation axis will remain fixed in space (conservation of angular momentum). This is the principle of the gyro-compass.

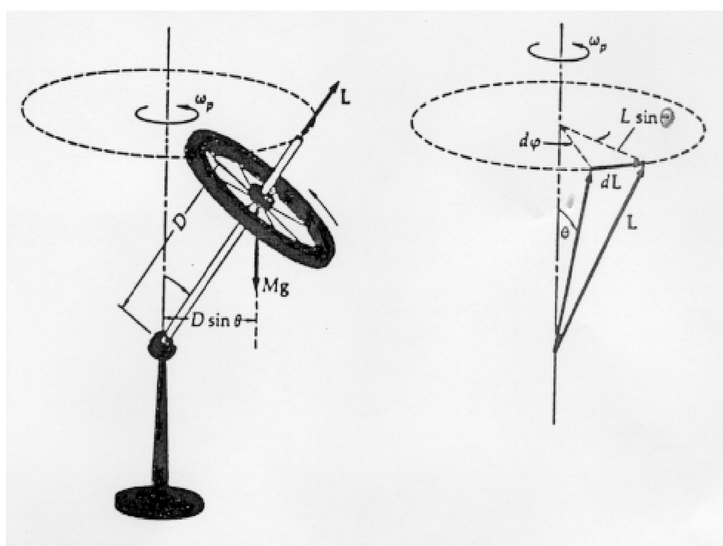

Figure 1: Gyroscopic precession.

If the gyro is now unbalanced (e.g., by the gravitational force at a distance D), the torque mg D sin θ causes a change in the angular momentum **L**. If the gyro is initially spinning with axial **L** (parallel to **ω**), then the change in **L** (d**L**) is perpendicular to **L**, and the axle moves in the direction of the torque. This motion is called precession. In the above figure, the torque **N** is in the horizontal plane and perpendicular to **L**. This torque results in a precessional angular velocity which may be found from the following consideration:

$$\frac{d\vec{L}}{dt} = \vec{N}, \tag{1}$$

where

$$|\vec{N}| = mg\ D \sin\theta$$

The angle dφ through which the axle moves is approximately

$$d\phi = \frac{dL}{L \sin\theta} = \frac{mg\ D \sin\theta\ dt}{L \sin\theta},$$

and the precessional angular velocity is

$$\omega_p = \frac{d\phi}{dt} = \frac{mg\ D}{L} = \frac{mg\ D}{I\omega}, \tag{2}$$

where ω is the spin angular velocity and I the moment of inertia. The assumption in the above derivation is that the total angular momentum is essentially $I\omega$. This condition can be achieved by having a large spin ω, or by having a large moment of inertia.

Nutation

If the gyroscope is also free to rotate in θ, gravity can exert a torque on the unbalanced gyroscope that is perpendicular to the direction of precession. If a gyroscope is released from an angle θ_1, rather than falling to the table, the gyroscope will stop at an angle θ_2, and rise back to θ_1, following one of the paths shown in Figure 2.

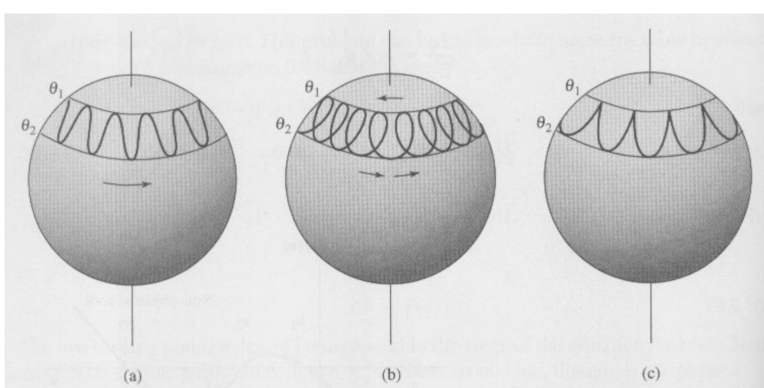

Figure 2: Nutation paths. (a) Initial velocity in the direction of precession. (b) Initial velocity counter to the direction of precession. (c) No initial velocity in φ.

This motion can be explained by examining the energy equation for the motion. The total energy of the gyroscope (minus the spinning energy in the disk, which is a constant when friction is ignored), is given by

$$E_{tot} = \frac{1}{2} I \left(\frac{\mathrm{d}\,\theta}{\mathrm{d}\,t} \right)^2 + \frac{1}{2} I \sin^2 \theta \left(\frac{\mathrm{d}\,\phi}{\mathrm{d}\,t} \right)^2 + \mathrm{m\,g\,D} \cos\theta \ , \qquad (3)$$

for which the derivation can be found in Reference 1. The two terms that depend on θ can be grouped together as a potential energy, V(θ):

$$V(\theta) = \frac{1}{2} I \sin^2 \theta \left(\frac{\mathrm{d}\,\phi}{\mathrm{d}\,t} \right)^2 + \mathrm{m\,g\,D} \cos\theta \qquad (4)$$

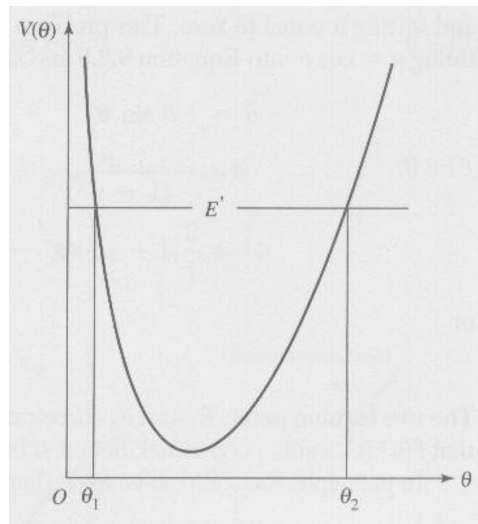

Figure 3: Nutation potential energy.

The $\sin^2\theta$ in this term means that V(θ) has a parabolic shape, when plotted against θ, as shown in Figure 3. The two maximum angles, θ_1 and θ_2, occur where this potential energy is equal to the total energy of the gyroscope. At these points, the potential energies will be equal, so θ_2 can be calculated from θ_1 by $V(\theta_2) = V(\theta_1)$.

The dependence of V(θ) on (dφ/dt) gives rise to the three paths of motion shown in Figure 2. In 2(c), the φ velocity is equal to the precessional velocity, ω_p, while 2(a) and 2(b) are the results of inducing non-zero initial velocities, with and against the precession velocity, respectively.

EQUIPMENT:

Gyroscope assembly
PASCO sensor equipment, computer, and DataStudio software
Add-on mass
Stabilizing rod with clamp
Pull strings
Long string for falling masses
Pulley on retort stand
Falling masses
Stopwatch
Calipers, ruler
Precision balance
Sighting tube

EXPERIMENT:

Using a large disk gyroscope, both the validity of Eq.(2) for precessional motion, and the predicted behaviour of nutational motion can be tested.

Warning: This gyroscope is a delicate apparatus! Be careful with it!

The gyroscope assembly is shown in Figure 4. The counterweights should exactly balance with the disk, but not the add-on mass.
Before beginning, ensure that the assembly is level on the table. Ensure that the apparatus does not freely rotate when rotated by hand to any location. If it does, follow the levelling instructions at the end of the lab. Also ensure that the counterweights are in the correct position to balance the apparatus at 90° to the table. Do not move the base once the gyroscope is level.

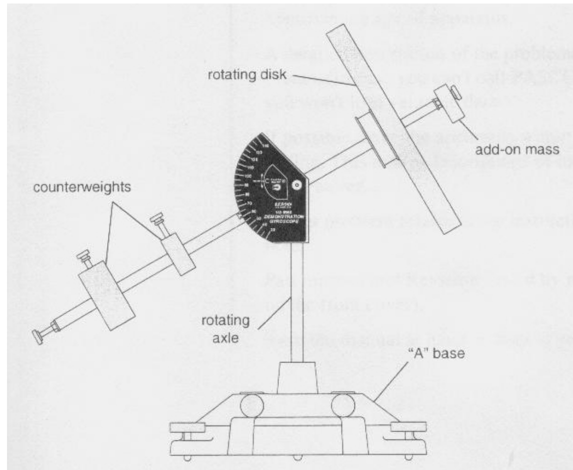

Figure 4: Gyroscope apparatus.

1) Measure the velocity of precession, and the angular velocity of the disk.
In order to validate Equation 2, the velocity of precession must be measured. The angular velocity must be simultaneously measured, to be used in the right-hand side calculation. Since the angular velocity will be different every time the experiment is performed, the easiest way to validate Equation 2 is to compare the product of the two varying quantities with the constants:

$$\omega_p \, \omega = \frac{m\,g\,D}{I} \tag{5}$$

In this way, the results of repeated experiments can be used together.

a) Mark a point on the disk with tape to identify the beginning of a rotation.
b) Holding the gyroscope apparatus at 90°, place the add-on mass on the end.
c) Start the disk spinning using one of the pull-strings.
d) While still holding the end of the gyroscope axle, use the sighting tube and a stopwatch to time about 10 revolutions to determine the initial angular velocity.
e) Let the axle go, and use DataStudio to determine the precessional velocity. The retort stand behind the gyroscope can be used to help keep the sensor's cord out of the way.
f) Friction can affect the angular velocity if the disk is spun for a long time, so the experiment should be performed quickly, or a measurement of the final angular velocity can be performed, to obtain an average.

2) Determine the values of m, g, and D.
a) The precision balance can be used to determine the weight of the add-on mass, while a ruler can be used to determine its distance from the centre of rotation. Look up a precise value of g for Guelph.

3) Determine the moment of inertia of the disk by calculation.
a) The weight of the disk was measured to be 17??±1 g (including the aluminium dowel). Determine the disk's moment of inertia using physical measurements.

4) Determine the moment of inertia of the disk by applying a known force.
a) The moment of inertia of the disk can also be found by accelerating the disk with a known torque. Clamp the gyroscope axle in place, as shown in Figure 5.
b) Put the retort stand with the pulley in position, and attach the long string to the gyroscope, and to a hanging mass, as shown in Figure 6.
c) Measure the angular acceleration using DataStudio, with appropriate error.

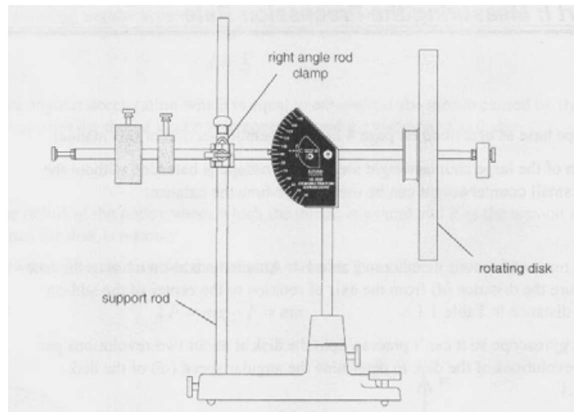

Figure 5: Gyroscope clamped in position.

d) Determine the friction impeding this acceleration.
i) The friction on the rotating axle is not negligible. Start with a very light mass, and determine the acceleration of a range of masses.
ii) Fit these data, and determine what mass is required to barely overcome friction.

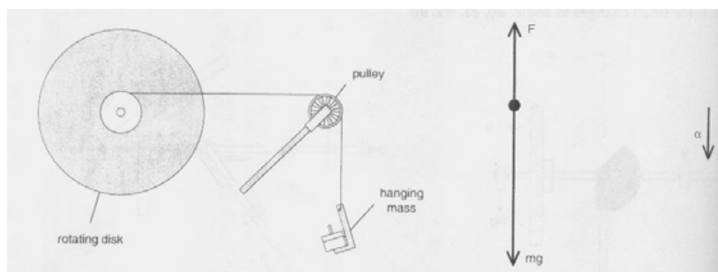

Figure 6: Setup for determination of the moment of inertia.

Factor this angular acceleration into your calculation of the moment of inertia.
e) Compare the moments of inertia found by the two methods. Which should be more accurate? Which appears to be more precise?

5) Compare your values for the left and right sides of Equation 6, taking uncertainties into account.

6) Nutation: Examine the paths of nutational motion.
a) Allowing the gyroscope to move freely again, raise the axle about 30° above horizontal. Spin the disk, and examine the paths of motion for different initial velocities using DataStudio. How do these paths compare to those predicted in Figure 2?

7) Verify the potential energy equation for nutational motion.

a) Relate the physical angle to that measured by DataStudio by holding the axle at a number of angles, and determining what numerical value DataStudio provides. Determine a linear equation to convert DataStudio values to physical angles.

b) Spin the gyroscope, and release the gyroscope from about 30° above the horizontal with NO initial velocity. By doing this, the precessional velocity can be used in the potential energy equation.

c) Use the measured value of θ_1 to predict the value of θ_2, and compare this to the measured value of θ_2.

Levelling the base:

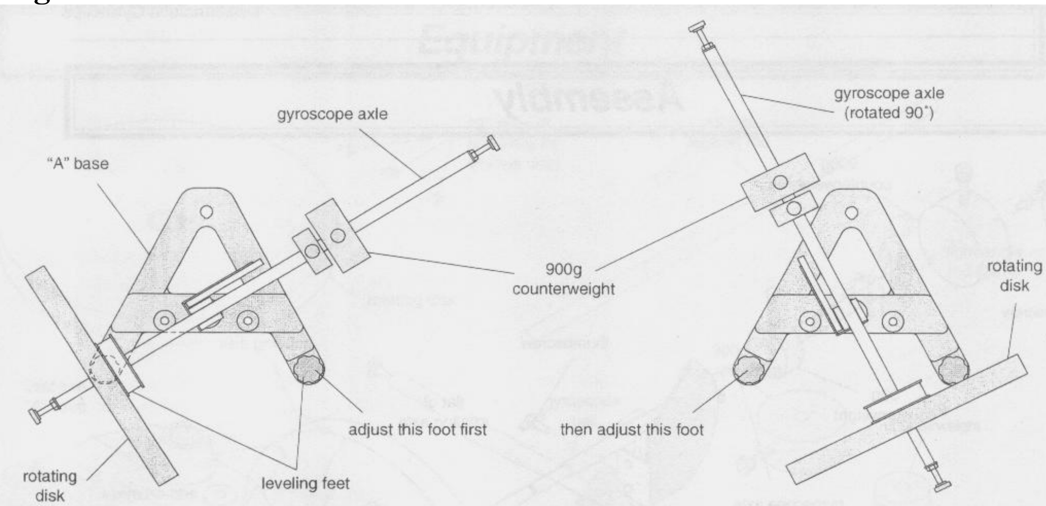

Figure 2: Leveling the Base

Leveling the Base

① Purposely make the apparatus unbalanced by moving the 900g counterweight towards the center.

② Adjust the leveling foot on one of the legs of the base until the gyroscope disk is aligned over the leveling foot on the other leg of the base. See Figure 2.

③ Rotate the gyroscope 90 degrees so the gyroscope axle is parallel to one side of the "A" and adjust the other leveling foot until the shaft will stay in this position. See Figure 2.

④ Adjust the position of the 900g counterweight until the gyroscope is balanced without the add-on mass. The 30g counterweight can be used to fine tune the balance.