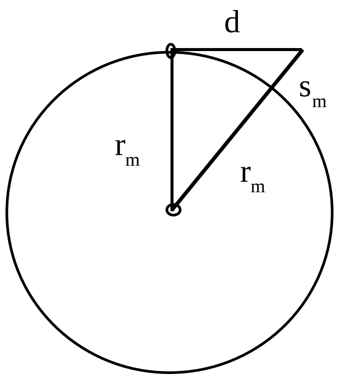
The Apple and the Moon

Worksheet on that 1/20 of an inch!



$r_m^2 + d^2 = (r_m + s_m)^2 = r_m^2 + 2r_m s_m + s_m^2 \approx r_m^2 + 2r_m s_m$ (why?)

hence $d^2 \approx 2r_m s_m$ and $s_m = \dfrac{d^2}{2r_m} =$ _____ inches

where s_m is the distance the moon falls back toward the earth in 1 sec.

Below we calculate d and r_m.

The Law of Inertia predicts a straight line path for the moon.
We can calculate the speed of the moon using "distance equals rate times time".

$$\text{moon speed} = \frac{2\pi r_m}{1\text{month}} = \frac{2\pi r_m}{27.32\text{days}} = \_\_\_\_\_ \text{ in/sec}$$

where r_m stands for the earth-moon distance of $3.84 \times 10^8 m$ = 239,000 mi = _____ inches.

Using $d = rt$ for one second we expect the moon to travel a distance of $d = (\quad)(1)$ = _____ inches.

Now, how does Newton's Universal Law of Gravitation predict how far the moon will fall back toward the earth in one second? (ie, what value did Newton's theory obtain for s_m?

Newton's law allows us to calculate the acceleration of an apple toward the earth using:

$$G = 6.67 \times 10^{-11} \text{ and } m_e = 5.98 \times 10^{24} \text{kg and } r_e = 6.38 \times 10^6 m$$

$$m_a a_a = G\frac{m_a m_e}{r_e^2} \Rightarrow a_a = 9.80\frac{m}{s^2} \approx 32\frac{ft}{s^2}$$

Similarly we obtain:

$$m_m a_m = G\frac{m_m m_e}{r_m^2} \Rightarrow a_m = G\frac{m_e}{r_m^2}$$

We can now calculate the acceleration of the moon, a_m, without G or m_e.

All we need is $r_e \approx 4000$ mi and $r_m \approx 240{,}000$ mi (actually 3970mi and 239,000mi).

$$\frac{a_m}{a_a} = \frac{a_m}{32\frac{ft}{s^2}} = \frac{G\frac{m_e}{r_m^2}}{G\frac{m_e}{r_e^2}} = \left(\frac{r_e^2}{r_m^2}\right) = \left(\frac{1}{60}\right)^2 \Rightarrow a_m = \frac{1}{3600}32\frac{ft}{s^2} = \_\_\_\_\_ \frac{in}{s^2}$$

Now we can use: $s_m = \dfrac{1}{2}at^2$ with t = 1 sec. to find $s_m =$ _____ *in.*

What is the percent error between the two s_m's?