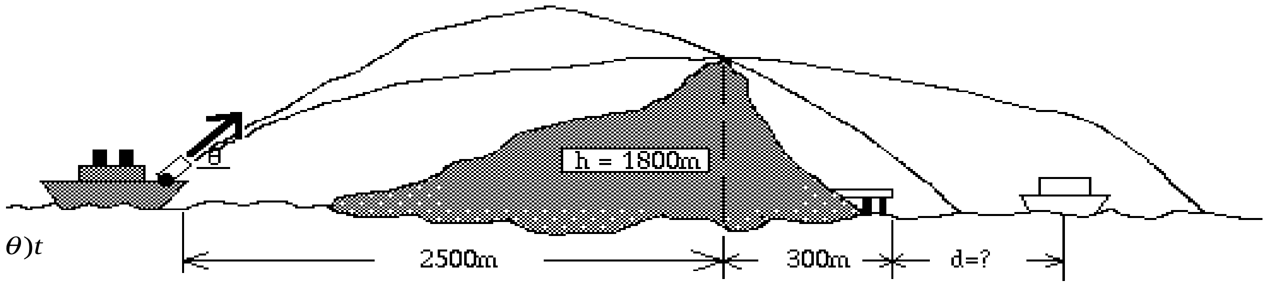
# THE BATTLESHIP PROBLEM (Serway & Faughn, College Physics p. 73, #51)




$$\begin{cases} x = (v_0 \cos\theta)t \\ y = -4.9t^2 + (v_0 \sin\theta)t \end{cases}$$

We need y > 1800m when x = 2500 and using $v_0 = 250\,{}^{m}\!/_{s}$ , we obtain

(i) $x = (250\cos\theta)t = 2500$ *and* (ii) $y = -4.9t^2 + (250\sin\theta)t > 1800$

$$or\ t_* = \frac{2500}{250\cos\theta} = \frac{10}{\cos\theta}\ and\ substituting\ in\ \text{(ii)} - 4.9\left(\frac{10}{\cos\theta}\right)^2 + 250\sin\theta\left(\frac{10}{\cos\theta}\right) > 1800$$

Dividing both sides of the last inequality by 100 we obtain

$$-4.9\sec^2\theta + 25\left(\frac{\sin\theta}{\cos\theta}\right) > 18 \text{ but this is a quadratic in } \tan\theta\,!$$

(using the trig identity : $1 + \tan^2\theta = \sec^2\theta$...)

$$-4.9(1 + \tan^2\theta) + 25\tan\theta > 18$$

$$-4.9 - 4.9\tan^2\theta + 25\tan\theta - 18 > 0$$

$$4.9\tan^2\theta - 25\tan\theta + 22.9 < 0$$

(dividing by 4.9 and using the quadratic formula...)

$$\tan^2\theta - 5.1\tan\theta + 4.67 < 0 \qquad \tan\theta = \frac{5.1 \pm \sqrt{7.33}}{2} = 3.9\ or\ 1.2$$

therefore we need to solve this inequality : $(\tan\theta - 1.2)(\tan\theta - 3.9) < 0$

(and since tangent is an increasing function for angles from 0 to 90 degrees...)

$1.2 < \tan\theta < 3.9$ is equivalent to :

$50.2° < \theta < 75.6°$

If $\theta_1 = 50.2°$, $\begin{cases} x = 160t \\ y = -4.9t^2 + 192t \end{cases}$

then $t_f = 39.2$

$Range = 6270m$

If $\theta_2 = 75.6°$, $\begin{cases} x = 62.2t \\ y = -4.9t^2 + 242t \end{cases}$

then $t_f = 49.4$

$Range = 3070m$

Recall the Range Theorem :

$$R = \frac{v_0^2 \sin 2\theta}{g} \text{ and notice that the range decreases after } \theta = 45°$$

Hence the range of the bombs is : 3070 < Range < 6270

Subtracting 2500 + 300 or 2800 to get the distance from the dock...

the "deadly range" or distance "d" from the dock is : 270 < d < 3470m

So...to be safe, move your little boat closer than 270m or further than 3470m!

(*By the way...*) *when* $x = 2500 = 160t$

$t_* = 15.6s$

*and* $v(t_*)$ *is positive*

*and the bomb is on the way up*

*when* $x = 2500 = 62.2t$

$t_* = 40.2s$

*and* $v(t_*)$ *is negative*

*and the bomb is on the way down*