Vectors

A **vector** is a 1, 2, or 3-dimensional concept with a **magnitude** and a **direction**.

ex/ Velocity is a vector concept.

In one dimension, we often picture motion along the mathematical x-axis.

we write:	magnitude	direction
5 m/s	5 m/s	positive-x
-5 m/s	5 m/s	negative-x

In two dimensions, vectors can be written in many different ways.

(a) mathematical **ordered pairs** in the xy-coordinate plane

we write:	magnitude	direction
(3,0) m/s	3 m/s	positive-x
(0,3) m/s	3 m/s	positive-y

(b) map directions

ex/ An airplane is flying with a velocity of:

we write:	magnitude	direction
161 km/h,NNW	161 km/h	NNW (North by Northwest)

(c) in trigonometry, the **direction angle**, θ, is defined as the angle measured counterclockwise (ccw) from the positive x-axis.

we write:	magnitude	direction
5 m/s,θ=53°	5 m/s	53° ccw from positive x-axis

(d) in navigation, compass **headings** or **bearings** are angles measured clockwise (cw) from North

we write:	magnitude	direction
25 knots,bearing of 90°	25 nautical miles/h	East

*Be careful with winds. Their headings are the direction from which they are blowing. Hence, a west wind blows east!

In three dimensions, it is usually best to use an xyz-coordinate system and **ordered triples**.

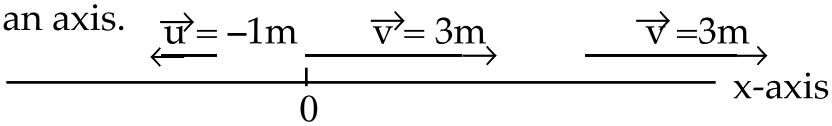
we write:	magnitude	direction
(-2,0,0) m/s	2 m/s	negative-x
(0,7,0) m/s	7 m/s	positive-y
(0,0,9) mi/h	9 mi/h	positive-z

Vectors

The **graph** or geometric representation of a vector (notation: $\vec{v}$) is an arrow or a directed line segment.

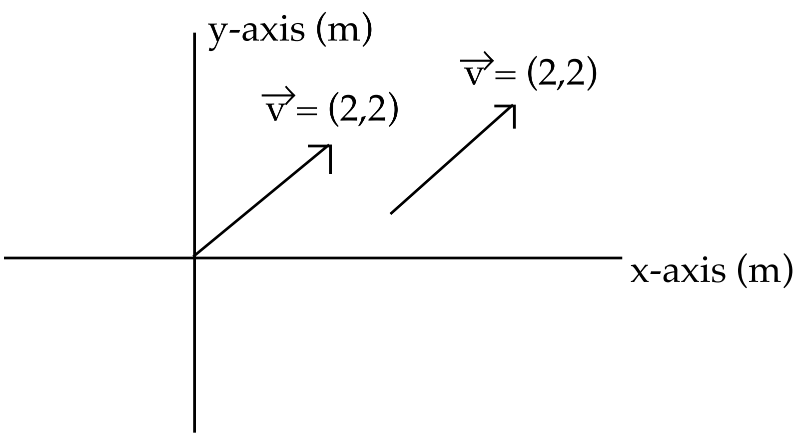
ex/ Displacement is a vector concept.

In one dimension, we can represent a displacement to the right and to the left on an axis.



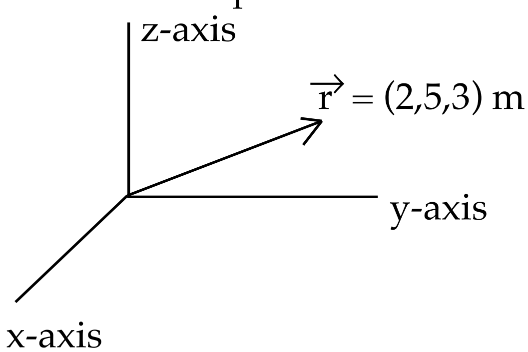
Notice that the two graphs on the right represent the same vector. They each have the same magnitude and the same direction. The middle graph represents a displacement from the origin and is said to be in **standard position**.

In two dimensions, the graphs may be angled up or downward.



Notice that the two graphs represent the same displacement vector, two meters to the right and two meters upward. (notation: The magnitude of the vector, $\vec{v}$, is written without the "→" symbol: $v = 2\sqrt{2}$ m) The direction angle or direction is 45° (ccw from the positive x-axis).

In three dimensions, we primarily use what is called a "righthand" xyz-coordinate system. An example of one is shown below.

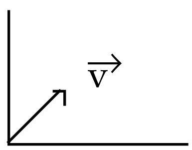


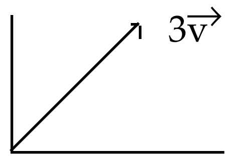
A **<u>unit vector</u>** is a vector whose magnitude is one unit. (notation: If $\vec{v} = (3,0)$, then the unit vector in the same direction would be written, $\hat{v} = (1,0)$.)

Certain unit vectors are the basis for our coordinate systems. In two dimensions, we write: $\hat{\mathbf{i}} = (1,0)$ and $\hat{\mathbf{j}} = (0,1)$. In three dimensions we write: $\hat{\mathbf{i}} = (1,0,0)$, $\hat{\mathbf{j}} = (0,1,0)$, and $\hat{\mathbf{k}} = (0,0,1)$.

<u>Scalar Multiplication</u> is a mathematical operation which is like a magnification of a vector. If k (a **scalar**) is a constant and $\vec{v} = (a,b)$, then $k\vec{v} = (ka,kb)$.

ex/ Let k=3 (magnification factor)
and let $\vec{v} = (1,1)$, then $3\vec{v} = (3,3)$ (a longer vector)

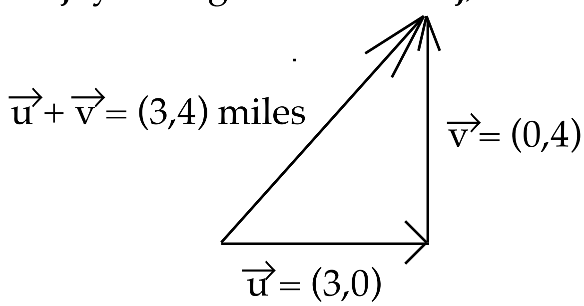




If k< 0 (negative), then the new vector direction is turned around (opposite).

ex/ Let k = -2 and let $\vec{r} = (3,4)$, then $-2\cdot\vec{r} = (-6,-8)$.

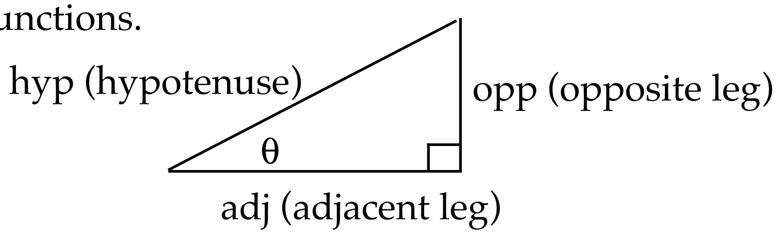
<u>Vector Addition</u> is the summing of two vector effects. If $\vec{u} = (3,0)$ miles and $\vec{v} = (0,4)$ miles are two displacement vectors, then $\vec{u} + \vec{v} = (3,4)$ miles represents the "sum" of these two separate displacements. We could also have written: $\vec{u} = 3\hat{\mathbf{i}}$ and $\vec{v} = 4\hat{\mathbf{j}}$ yielding $\vec{u} + \vec{v} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$, miles.



One tends to see first a displacement to the right followed by a displacement upward. With vectors, both parts of the **sum** or **resultant** vector may occur simultaneously.

ex/ Force is a vector concept. Let $\vec{F}_1 = 3$ lb pull, in the positive-x direction, and let $\vec{F}_2 = 4$ lb pull, in the positve-y direction. Now $\vec{F}_1 + \vec{F}_2 = 5$ lb pull, at a 53° angle with the x-axis. Here both forces were acting simultaneously to produce a net effect.

Recall the right triangle definitions of the **<u>sine</u>**, **<u>cosine</u>**, and **<u>tangent</u>** trigonometry functions.



sin θ = opp/hyp
cos θ = adj/hyp
tan θ = opp/adj or sin θ/cos θ

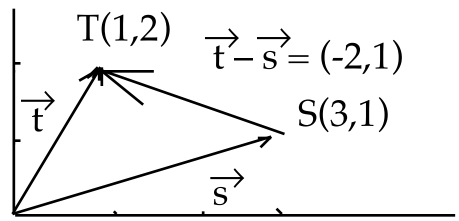
Vectors

Problem Set 1.0

1. We define **speed** to be the magnitude of velocity. If $\vec{v} = (5,12)$ m/s is the "take-off" velocity of a long jumper, what is it's initial speed?
2. Winnie the Pooh's balloon has burst. As he falls, he reaches a "terminal velocity" of -15 $\hat{\mathbf{k}}$, m/s. What is Pooh's speed when he hits the mud puddle?
3. Broom-Hilda's top speed is 40 km/h. If she is flying at this "air speed" into a "headwind" of -50 km/h, what is her net velocity?
4. Now Broom-Hilda is flying with an air speed of 30 km/h at a heading of 90°. A west wind is blowing with a speed of 15 km/h. What is Broom-Hilda's resultant velocity?
5. Garfield has just booted Odie giving him an initial velocity of (3,4,12) m/s. What is Odie's initial speed?
6. Garfield is working on a 2-dimensional trigonometry problem. As an experimental physicist, he sends his friend, Odie, from (0,0) with a speed of 8 m/s and a direction angle of 37°. What point will Odie be passing when Garfield's stopwatch reads 7 sec?
7. Calvin and Hobbes are tugging on an atomic-powered ray gun. Calvin pulls with a force of (2,2) N (newtons) and Hobbes pulls with a force of (-5,2) N. What is the magnitude of the net force on the gun, and in what direction does it move?

If $\vec{v}$=(a,b) and $\vec{u}$=(c,d), then the **<u>difference vector</u>** between $\vec{u}$ and $\vec{v}$, $\vec{u}-\vec{v}$, is (c-a,d-b)

There can be a confusion with the use of ordered pairs to represent vectors since in elementary algebra, ordered pairs were used exclusively to designate points in a plane. Suppose, for example, we start at a point, S(3,1), and end up at a point, T(1,2). The **displacement vector** would also be represented by an ordered pair, (-2,1). This vector is obtained by subtraction and is an example of a **difference vector**. In fact, we can associate **position vectors**, $\vec{s}$ and $\vec{t}$, with the points, S and T. Position vectors are always drawn in standard position bound to the origin, while displacement or difference vectors are "free" vectors which can be graphed anywhere. Notice how $\vec{t}-\vec{s}$ was drawn as a free vector not bound to the origin.



Be careful, vector graphs do not usually represent points or paths of motion.

ex/ If $\vec{r}_1$(6,1) represents our initial position, and $\vec{r}_2$(9,1) represents our final position, then the displacement from $\vec{r}_1$ to $\vec{r}_2$ is (3,0). This is also called the **<u>change in position</u>** (notation: $\Delta\vec{r}$).

ex/ If $\vec{v}_1$ (-3,0)m/s is our initial velocity, and $\vec{v}_2$(-7,0)m/s is our final velocity, then the difference between $\vec{v}_2$ and $\vec{v}_1$ is (-4,0)m/s. This is also called the **<u>change in velocity</u>** (notation: $\Delta\vec{v}$). (This difference vector is not a displacement vector.)

ex/ If x_1= 4m and x_2=1m are initial and final positions, respectively, then x_2-x_1 or Δx = -3m. (Notice that in one dimension, the vector symbol "→" is usually dropped.)

The **<u>zero vector</u>** has zero magnitude and has any direction. We write $\vec{0}$ = (0,0) or (0,0,0).

Two nonzero vectors, $\vec{u}$ and $\vec{v}$, are **<u>parallel</u>** iff $\vec{u} = k\vec{v}$ for some k∈ℜeals.

Two nonzero vectors are parallel iff one is a **scalar multiple** of the other.

ex/ (3,1,-6) and (6,2,-12) are parallel vectors. Try k=2.

ex/ (-3,4) and (3,-4) are parallel vectors with opposite directions. Try k= -1.

Two nonzero vectors are **<u>perpendicular</u>** or **<u>orthogonal</u>** iff they form a right angle when drawn in standard position bound to the origin.

ex/ (3,4) and (4,-3) are orthogonal vectors.

ex/ Also, (3,4) ⊥ (-4,3).