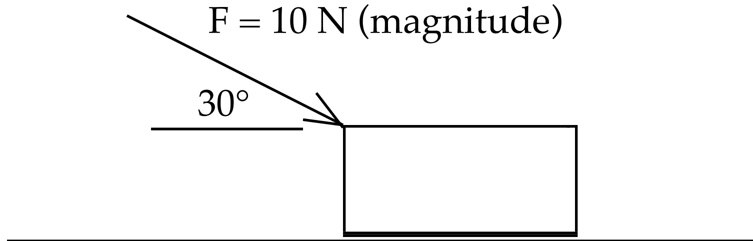
Vectors

One of the most important manipulations made with a vector is to express the vector in terms of **components**. Most often we express or **resolve** a vector in terms of the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ unit vectors.

ex/ Let $\vec{r}$ have a magnitude of 6m with a direction angle of 30°. We can write: $\vec{r} = 3\sqrt{3}\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$, m or $\vec{r} = (3\sqrt{3}, 3)$, m. Now add this vector to (1,2) and to (2,-7) simultaneously. Addition and subtraction can be easy!

ex/ What component of the force pushing the block (shown below) is to the right? What part or component of the force is actually pushing the block downward into the floor?



Components can refer to the real number part of an ordered pair. This is also called the **<u>scalar component</u>**. Components can also include direction and hence refer to the **<u>vector component</u>** of the vector being discussed.

ex/If $\vec{v} = (3,4)$ m/s, we say that the scalar component of $\vec{v}$ in the x-direction is 3 m/s. We say that the vector component of $\vec{v}$ in the x-direction is $3\,\hat{\mathbf{i}}$, m/s. (This is not an important distinction to make, but your math teacher may wish to make it!)

Now we come to a strange-looking operation that is used in more advanced math and physics courses. The **<u>dot product</u>** of two vectors, $\vec{u} = (a,b)$ and $\vec{v} = (c,d)$, turns out to be a scalar (real number). We write:

$$\vec{u} \cdot \vec{v} = ac+bd \text{ (not a vector!)}$$

In trigonometry you will prove that $\vec{u} \cdot \vec{v} = u \cdot v \cos\theta$ where θ is the angle between the vectors. (Remember that u and v represent magnitudes, and hence, $u \cdot v$ is the normal product of two numbers that you are used to!)

ex/ What is the dot product of (3,4) and (4,-3)?

Method 1: $3 \cdot 4 + 4 \cdot (-3) = 0$ Method 2: $5 \cdot 5 \cos 90° = 0$

Notice that two nonzero vectors are <u>orthogonal iff their dot product is zero</u>.

Finally (!) there is another, even stranger operation called the **<u>cross product</u>** of two vectors. Here we do get a vector answer.

Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ then $\vec{u} \times \vec{v} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \end{vmatrix}$ Egads!